NPS-OR-98-005

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Suppression of Enemy Air Defense (SEAD) as an Information Duel

by

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August 1998

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Prepared for: Space-C2-Information Warfare, Strategic Planning Office, N6C3
2000 Navy Pentagon, Washington, DC 20350-2000

Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943

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This report was prepared for and funded by Space-C2-Information Warfare, Strategic Planning Office, N6C3, 2000 Navy Pentagon, Washington, DC 20350-2000, and the Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943.

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REPORT DOCUMENTATION PAGE

Form approved

OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	August 1998	Technical

4. TITLE AND SUBTITLE	5. FUNDING
Suppression of Enemy Air Defense (SEAD) as an Information Duel	N0001498WR30081

6. AUTHOR(S)	
Donald P. Gaver and Patricia A. Jacobs	

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Naval Postgraduate School Monterey, CA 93943	NPS-OR-98-005

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
Space-C2-Information Warfare, Strategic Planning Office, N6C3 2000 Navy Pentagon, Washington, DC 20350-2000 Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943	

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words.)

Mathematical models are furnished for a situation in which Red missile-shooting air defense forces engage attacking Blue striking aircraft. Red may *either* employ extensive electronic emissions when firing at Blue, which renders it more effective but more vulnerable to Blue counterattack, *or* operate more covertly (less emission) which makes it less effective but also less vulnerable to Blue retaliation. Simple decision rules dictate the optimal, or near-optimal, policy for Red to follow. This then indicates to Blue her maximum opposition. In simple cases the optimal Red strategy is the same for both a deterministic and (quite different) stochastic model.

14. SUBJECT TERMS	15. NUMBER OF PAGES
air defense, SEAD, combat models, stochastic duels	31
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5800

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18

SUPPRESSION OF ENEMY AIR DEFENSE (SEAD) AS AN INFORMATION DUEL

Donald P. Gaver
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1. Introduction: (Red) Enemy Air Defense and Its (Blue) Suppression (SEAD)

A group of Blue striking aircraft (the Attackers) is entering a region, denoted as $\mathcal{R}$, to attack Red assets therein. Within the region are a number of Red air-defense installations, generically Enemy Air Defense (EAD) shooter subsystems. These, the Defenders, have the capacity to jam and shoot down (via ground-to-air missiles, or, eventually, advanced directed energy weapons), the Blue Attackers. They are coordinated by communication linkages, elements of which may be subject to attack physically and by Information Operations (IO) techniques.

To oppose the above, i.e. conduct *suppression of enemy air defenses* (SEAD), the Blue force can select from various assets and tactics. This report discusses some optional combinations of systems and tactics in terms of simple state-space models, both deterministic and stochastic. The stochastic models are Markov processes that can be solved explicitly in the present circumstances, or, if desired, as object-oriented simulation models with stochastic features at a later stage (this latter step is not taken here). First we specify some elements of the interplay between Attackers and Defenders. Our presentation suggests several alternative/optional models, all of which have the feature that information acquisition can be both beneficial and harmful, and that a balance can be struck. Because of the stripped-down approach taken it is possible to explicitly characterize "optimal" strategies in simple form. For further, more extensive, work in the present area see Glazebrook, Gaver, and Jacobs (1998).

2. Model D-1: An Exploratory Deterministic 2-Sided Model for SEAD

2.1. The Setting and Modeling Approach

We study a simplified version of an information duel between Blue Attackers (e.g. EA6B a/c equipped with air-to-ground HARM missiles) and Red (Area) Defenders, equipped with anti-air missiles. The latter are arranged in defense of a region, and are composed of a system of Early Warning Radars, $R_{EW}(t)$ in number at time t, and a further system of radar-equipped anti-air missile shooters, called in *our* jargon Full Houses, and in number $R_A(t)$ at t.

Scenario

The scenario is this: at time $t = 0$

(a) A force of $B_U(0)$ Blue Attackers arrives, or is initially present, at the edge of the detection envelope of the Red EW force, of size/capability R_{EW}. The number of Blues present but undetected at t (in undetected state) is $B_U(t)$ for any $t > 0$.

(b) Blues are detected at a rate in time that reflects the number of Red EW units available, and the number of Blues undetected at time $t > 0$. The number detected at time t (in the detected state) is denoted by $B_D(t)$.

(c) Blues detected (by Red) units are placed on a Red Full House system (shooter) target list, regarded as a service/queuing system with $R_A(t)$ "servers" (i.e. missile shooters). A *service time* is a tracking time that terminates with a shot.

(d) Red shooters have two modes of operation:

(d-1) extensive radar emissions, in which case the probability of defensive missile kill of a Blue is relatively high, but retaliatory response by some Blues is quite likely, and also relatively effective;

(d-2) minimum radar emissions, in which case Blues are less vulnerable (smaller kill probability of R on B), *but* Red Full House units are also less detectable.

Here next are dynamic equations to describe the above interchange.

2.2. Dynamic Equations

$$\frac{dB_U(t)}{dt} = \underbrace{\lambda(t)}_{\substack{\text{Blue arrival}\\ \text{rate at } t}} - \underbrace{\xi_{EW} B_U(t) R_{EW}(t)}_{\substack{\text{Detection rate of Blue}\\ \text{by Red Early Warning}}} \tag{2.1}$$

This equation describes the evolution of the undetected Blue population in the area at t, $B_U(t)$. It does not model saturation of the Red Early Warning system or its targeting and attrition; Blues undetected become detected in proportion to their number and the number of Red EW facilities, which for the present is $R_{EW}(t) = R_{EW}(0) = R_{EW}$. Next,

$$\frac{dB_D(t)}{dt} = \underbrace{\xi_{EW} B_U(t) R_{EW}(t)}_{\substack{\text{Detection rate of Blue}\\ \text{by Red Early Warning}}} - \underbrace{v_{RB} R_A(t) \frac{B_D(t)}{1+B_D(t)} \left(\theta_{RI} P_{RI} + \theta_{RQ} P_{RQ}\right)}_{\substack{\text{Attrition rate of detected Blues by Red actives,}\\ \text{either emitting extensively or minimally}}}. \tag{2.2}$$

The complete Blue attrition term $v_{RB} R_A(t) \left(\frac{B_D(t)}{1+B_D(t)} \right)$ represents the rate at which Red shooters complete acquisition and tracking of detected Blues; the component term $\left(B_D(t)/(1+B_D(t))\right)$ represents saturation of the Red forces by Blues in queue (on target list) to be shot: here if $B_D(t)$ much exceeds unity then Red forces can only complete tracking at a rate proportional to their own (current) force size; see Filipiak (1988), and Gaver and Jacobs (1998). Saturability at a larger value can be adjusted by adding a parameter, and provisions for loss of Red track on Blue can likewise be made in the model; Blue decoys can be added. The term $(\theta_{RI} P_I + \theta_{RQ} P_Q)$ represents the kill probability of a Red shooter system that *either* chooses to shoot using extensive emission (probability of this choice is θ_{RI}), in which case the kill probability is P_{RI}; *otherwise* Red utilizes a "quiet" mode, i.e. with minimal emission (probability of this choice is $\theta_{RQ} = 1 - \theta_{RI}$), so as a result the kill probability is P_{RQ}. It is anticipated that P_{RI} is greater than P_{RQ}. Although mode I leads to higher kill probability it also exposes the Red shooter to Blue detection and more effective retaliation. The parameter θ_{RI} is thus a Red decision variable. We shall furnish simple rules for choosing its value.

For Red actives (Full House systems), $R_A(t)$, we stipulate the following.

$$\frac{dR_A(t)}{dt} = \underbrace{-\left(v_{RB} R_A(t) \frac{B_D(t)\theta_{RI}}{1+B_D(t)} \right) \cdot (B_D(t) + B_U(t)) v_{BR} P_{BRI}}_{\substack{\text{Rate at which extensively emitting Red} \\ \text{leads to retaliatory kills by Blue}}}$$
$$\underbrace{-\left(v_{RB} R_A(t) \frac{B_D(t)\theta_{RQ}}{1+B_D(t)} \right) \cdot (B_D(t) + B_U(t)) v_{BR} P_{BRQ}}_{\substack{\text{Rate at which minimally emitting Red response} \\ \text{leads to retaliatory kills by Blue}}} \tag{2.3}$$

Note that in (2.2) and (2.3) detected Blues are immediately targetable in this model; a realistic delay-prone communication network is not explicitly modeled here. The first term, $\left(v_{RB} R_A(t) \left(\frac{B_D(t)\theta_{RI}}{1+B_D(t)} \right) \right)$, represents the (saturable) rate at which the current Red force terminates preliminary tracking and emits extensively while prosecuting (probability θ_{RI}) Blue targets; this rate translates into a rate of counter-fire proportional to *all* live Blue forces ($B_D(t) + B_U(t)$); kill/attrition of the extensively-emitting (illuminating) Red by those Blue forces is at rate $v_{BR}P_{BRI}$. The subsequent term is the same as the last, but accounts for the occasions on which Red emits minimally ("is quieter") and hence is killed at a smaller rate, $v_{BR}P_{BRQ}$. Although a mixed policy is available, and can well be time and state-dependent, it may turn out that a Red policy will be to set θ_{RI} (hence θ_{RQ}) to *either* one or zero; i.e. adopt a pure strategy. An occasion when this is so follows.

2.3. Analysis

Suppose the "combat clock" is started at $t = 0$, with all Blues assigned for SEAD initially present at that time. Equations (2.1) – (2.3) can be explicitly analyzed in closed form if $\lambda(t) = 0$ and $R_{EW}(t) \equiv R_{EW}$, a constant. The solution to (2.1) is

$$B_U(t) = B_U(0)\exp\{-\xi R_{EW} t\}. \tag{2.4}$$

Let
$$\overline{\theta}_R = (\theta_{RI} P_{RI} + \theta_{RQ} P_{RQ}) v_{RB}$$

and $$\overline{\theta}_B = (\theta_{RI} P_{BRI} + \theta_{RQ} P_{BRQ}) v_{BR}.$$

Adding equation (2.1) and (2.2) results in

$$\frac{d\overline{B}(t)}{dt} = -R_A(t) \frac{B_D(t)}{1+B_D(t)} \overline{\theta}_R \tag{2.5}$$

where $\overline{B}(t) = B_U(t) + B_D(t)$; of course $\overline{B}(0) = B_U(0)$.

Equation (2.3) can be rewritten as

$$\frac{1}{\overline{\theta}_B} \frac{dR_A(t)}{dt} = -R_A(t) \frac{B_D(t)}{1+B_D(t)} \overline{B}(t). \tag{2.6}$$

Divide equation (2.6) by (2.5), which results in

$$\frac{dR_A(t)}{d\overline{B}(t)} = \frac{\overline{\theta}_B}{\overline{\theta}_R} \overline{B}(t). \tag{2.7}$$

Thus,

$$\overline{B}(t) d\overline{B}(t) = \frac{\overline{\theta}_R}{\overline{\theta}_B} dR_A(t). \tag{2.8}$$

Integrating results in

$$\left[\overline{B}^2(t) - \overline{B}^2(0)\right] = 2\frac{\overline{\theta}_R}{\overline{\theta}_B}\left[R_A(t) - R_A(0)\right]. \tag{2.9}$$

Notice that the solution is essentially parameterized by an exchange ratio, $\overline{\theta}_R / \overline{\theta}_B$. Further, Blue may have the advantage since Red is vulnerable while it is prosecuting targets.

Since $\lambda(t) = 0$, if $t \to \infty$, then either $\lim_{t\to\infty} \overline{B}(t) = 0$ or $\lim_{t\to\infty} R_A(t) = 0$. In fact, if $\overline{B}(0) > \sqrt{2\frac{\overline{\theta}_R}{\overline{\theta}_B} R_A(0)}$, then

$$\overline{B}(\infty) = \sqrt{\overline{B}^2(0) - 2\frac{\overline{\theta}_R}{\overline{\theta}_B} R_A(0)} \text{ and } R_A(\infty) = 0; \tag{2.10,a}$$

Blue wins, killing all Red AD units/shooters; if $\overline{B}(0) < \sqrt{2\frac{\overline{\theta}_R}{\overline{\theta}_B} R_A(0)}$, then $\overline{B}(\infty) = 0$ and

$$R_A(\infty) = R_A(0) - \frac{1}{2}\frac{\overline{\theta}_B}{\overline{\theta}_R}\overline{B}^2(0); \tag{2.10,b}$$

here Blue loses all forces, with Red AD survivors available for countering later attacks.

It is always to the advantage of Red to maximize the exchange ratio $\overline{\theta}_R/\overline{\theta}_B$: doing so either maximizes Blue's losses if $\overline{B}(0)$ is sufficiently large, or maximizes Red's survivors. Differentiation of the exchange ratio shows that

$$\begin{aligned} &\textit{Red should emit extensively } (\theta_{RI} = 1) \text{ if} \quad && \frac{P_{RI}}{P_{BRI}} > \frac{P_{RQ}}{P_{BRQ}} \\ &\textit{or, equivalently, } \text{if} && \frac{P_{RI}}{P_{RQ}} > \frac{P_{BRI}}{P_{BRQ}} \end{aligned} \tag{2.11}$$

Otherwise, Red illumination is held to a minimum, i.e. in Q-state. See Section 3 for the surprising reappearance of the above rule in the context of a seemingly quite different, stochastic model, context.

In words, Red should emit extensively if her relative advantage from so doing exceeds the *relative advantage* to Blue from Blue's capability to profit from/capitalize on Red's use of extensive emission. It is noteworthy that in this model the optimal strategy for Red holds regardless of the value of v_{RB}, Red's attrition rate on Blue; nor is there dependence on ξ_{EW}, the rate of detection of the Blues by the Red EW system. A more subtle model would represent Blue and Red reactions to actual occurrences, necessarily modeled stochastically interacting stochastically modeled; this step is postponed.

Blue SEAD planners can clearly make use of the above for planning purposes, i.e. to size an attacking force approximately. In a following section the same basic conclusion is deduced from a simple stochastic model.

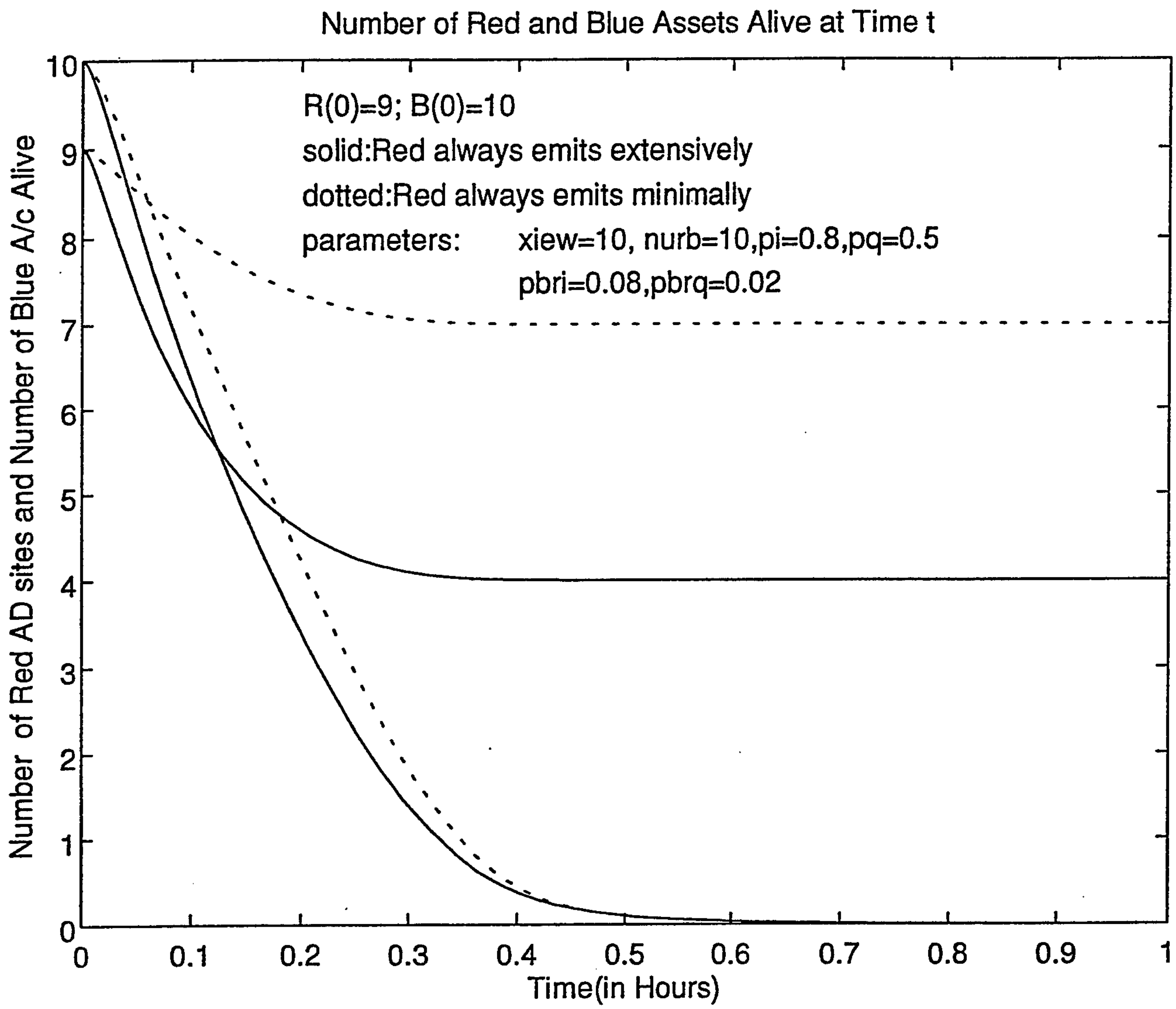
Figures 1-3 display the numbers of Red and Blue alive assets as a function of time. In Figure 1, $\xi_{EW}R_{EW}(t) = 10$ per hour; $v_{RB} = 10$ per hour (service rate for each Red); $P_{RI} = 0.8$ (probability extensively-emitting Red kills a Blue a/c); $P_{RQ} = 0.5$ (probability minimally-emitting Red kills a Blue a/c); $v_{BR}P_{BRI} = 0.08$ per hour (rate at which an extensively-

emitting Red is killed by a Blue a/c); and $v_{BR}P_{BRQ} = 0.02$ per hour (rate at which a minimally-emitting Red is killed by a Blue a/c). Figure 1 compares the numbers of alive Red and Blue assets as a function of whether or not Red is always emitting extensively or always emitting minimally. Red emitting minimally results in fewer casualties to itself but it takes a longer time to kill specified numbers of Blues. In Figure 2, $v_{BR}P_{BRI} = 0.05$, with the other parameters the same. Comparing the extensively-emitting cases of Figures 1 and 2, more Reds survive and any specified number of Blues are killed sooner for $v_{BR}P_{BRI} = 0.05$.

In Figure 3, the Reds are always emitting minimally, $v_{BR}P_{BRQ}= 0.05$, and the other parameters are as in Figures 1 and 2. Figure 3 displays the numbers of Red and Blue assets for differing values of the initial number of Blue a/c. Note that in the present situation the number of Blue aircraft needed to kill all the Red AD sites is more than twice the number of AD sites. An increase in Blue lethality should have great leverage.

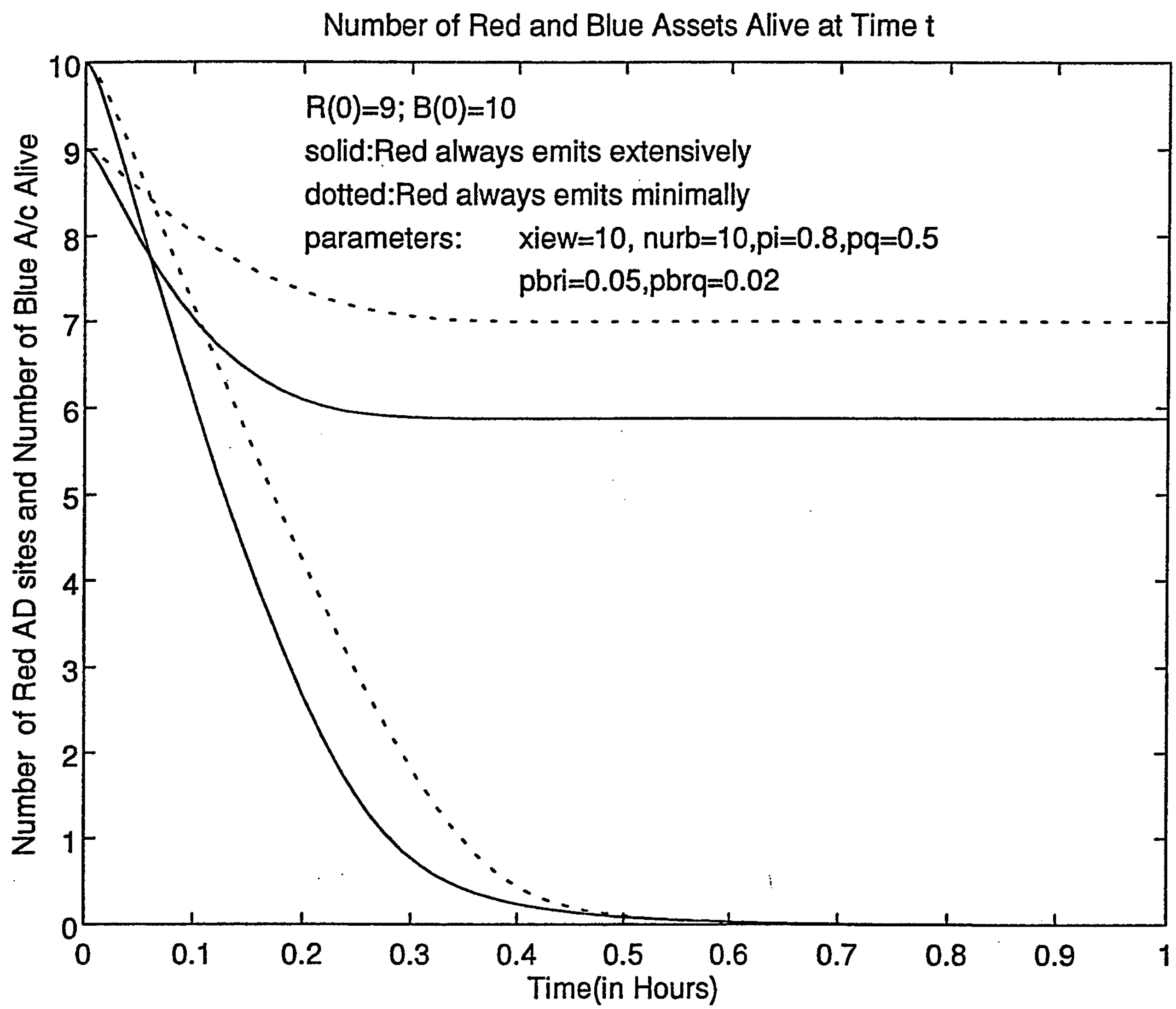
3. Model S-1: Elementary Stochastic Duel Between One (Red) AD System and a Succession of Blue Suppressors

Suppose a *single* Red (E)AD unit is in opposition to a Blue force intent on removing this obstacle. The Blue force might be platforms that are HARM-launchers devoted to suppressing enemy air defense (SEAD); they come within (Red) range so as to have better access to the target, but in so doing expose themselves to attrition. Our model yields explicit formulas for assessing attrition tradeoffs in the present simple setting.



Number of Red and Blue Assets Alive at Time t
R(0)=9; B(0)=10
solid:Red always emits extensively
dotted:Red always emits minimally
parameters: xiew=10, nurb=10,pi=0.8,pq=0.5
pbri=0.08,pbrq=0.02
Number of Red AD sites and Number of Blue A/c Alive
Time(in Hours)
0
1
2
3
4
5
6
7
8
9
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Figure 1



Number of Red and Blue Assets Alive at Time t
R(0)=9; B(0)=10
solid:Red always emits extensively
dotted:Red always emits minimally
parameters: xiew=10, nurb=10,pi=0.8,pq=0.5
pbri=0.05,pbrq=0.02
Number of Red AD sites and Number of Blue A/c Alive
Time(in Hours)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
9
8
7
6
5
4
3
2
1

Figure 2

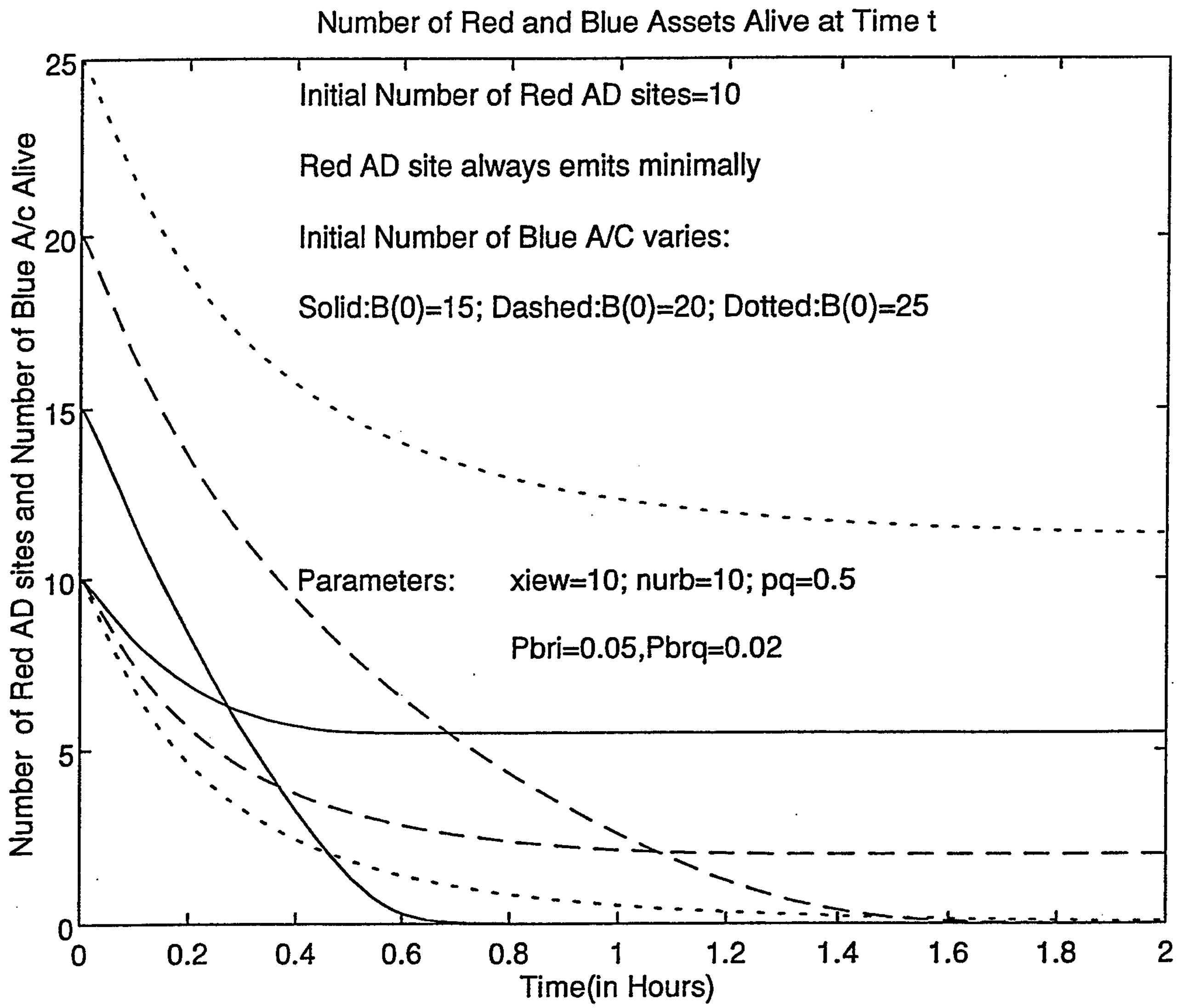

Number of Red and Blue Assets Alive at Time t
Initial Number of Red AD sites=10
Red AD site always emits minimally
Initial Number of Blue A/C varies:
Solid:B(0)=15; Dashed:B(0)=20; Dotted:B(0)=25
Parameters: xiew=10; nurb=10; pq=0.5
Pbri=0.05,Pbrq=0.02
Number of Red AD sites and Number of Blue A/c Alive
Time(in Hours)
0
5
10
15
20
25
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 3

Model Functionality

(a) Each time Red fires it is in *quiet mode* (emitting minimally) with probability θ_{RQ}, and otherwise in *extensive emitting/radiating mode* with probability $\theta_{RI} = (1 - \theta_{RQ})$. Here θ_{RI} is a possible decision variable; see Section 2.

(b) Given that Red is in the extensive emission mode (is emitting), Red is killed by a Blue with probability P_{BRI}. If it is in the quiet (minimally-emitting) mode, Red is killed with probability P_{BRQ}.

(c) Given that Red is in extensive emission mode, it kills a Blue opponent with P_{RI}; if Red is in quiet mode its Blue-kill probability changes (presumably drops) to P_{RQ}.

(d) Assume shots occur one at a time: Red fires at some Blue, and a Blue responds; this is an elementary transaction. These are repeated until Red is killed; in the meantime many Blues may be killed.

Questions:

(3.1) How many shots does Red complete, and how many Blues are killed, before Red is eliminated (by a successful shot; cumulative damage is not modeled).

(3.2) What is a good ("optimal"!) strategy for Red to follow so as to decimate the Blue force as extensively as possible before itself being eliminated?

Let K_B be the random number of Blues killed before Red is killed. Then

$$K_B = \begin{cases} 0 & \text{with probability } \theta_{RQ}(1-P_{RQ})P_{BRQ} + \theta_{RI}(1-P_{RI})P_{BRI} \\ 1 & \text{with probability } \theta_{RQ}P_{RQ}P_{BRQ} + \theta_{RI}P_{RI}P_{BRI} \\ 0+K_B' & \text{with probability } \theta_{RQ}(1-P_{RQ})(1-P_{BRQ}) + \theta_{RI}(1-P_{RI})(1-P_{BRI}) \\ 1+K_B' & \text{with probability } \theta_{RQ}P_{RQ}(1-P_{BRQ}) + \theta_{RI}P_{RI}(1-P_{BRI}) \end{cases} \tag{3.1}$$

where K_B' is a random variable having the same (unconditional) distribution as K_B: it is the result of "starting over".

Now take conditional expectations as in (3.1) to find

$$E[K_B] = 1 \cdot \left[\theta_{RQ} P_{RQ} + \theta_{RI} P_{RI}\right] + \left[\theta_{RQ}(1 - P_{BRQ}) + \theta_{RI}(1 - P_{BRI})\right] E\left[K_B'\right]$$

or

$$E[K_B] = \frac{\left[\theta_{RQ} P_{RQ} + \theta_{RI} P_{RI}\right]}{1 - \left[\theta_{RQ}(1 - P_{BRQ}) + \theta_{RI}(1 - P_{BRI})\right]}$$

$$= \frac{\theta_{RQ} P_{RQ} + \theta_{RI} P_{RI}}{\theta_{RQ} P_{BRQ} + \theta_{RI} P_{BRI}} \tag{3.2}$$

since $E\left[K_B'\right] = E[K_B]$.

If $\theta_{RQ} = 1$, then

$$E[K_B] \equiv E\left[K_B(Q)\right] = \frac{P_{RQ}}{P_{BRQ}}. \tag{3.3,a}$$

If $\theta_{RI} = 1$, then

$$E[K_B] \equiv E\left[K_B(E)\right] = \frac{P_{RI}}{P_{BRI}}. \tag{3.3,b}$$

If $\dfrac{P_{RQ}}{P_{BRQ}} > \dfrac{P_{RI}}{P_{BRI}}$, then

$$E[K_B] = \frac{\theta_{RQ} P_{RQ} + \theta_{RI} P_{RI}}{\theta_{RQ} P_{BRQ} + \theta_{RI} \delta_E} \leq \frac{P_{RQ}}{P_{BRQ}} \qquad \text{for } 0 \leq \theta_{RQ} \leq 1$$

For Red, the policy that maximizes the expected number of Blues killed before it is eliminated is determined by a simple transaction kill ratio

$$\begin{aligned} &\textit{Emit extensively} \text{ if} \quad \frac{P_{RI}}{P_{BRI}} > \frac{P_{RQ}}{P_{BRQ}} \\ &\textit{Emit minimally} \text{ if} \quad \frac{P_{RQ}}{P_{BRQ}} > \frac{P_{RI}}{P_{BRI}} \end{aligned} \tag{3.4}$$

(Extensive and minimal emissions are equally effective if equality holds.)

This is exactly the condition (2.11) found for the deterministic model.

Model S-2:

Assume there are i types of Blue targets: $i = 1, \dots I$. Let $P_{RQ}(i)$ (respectively $P_{RI}(i)$) be the probability of a quiet (respectively extensive emitter) Red killing a type i Blue target. Let α_i be the probability a Blue target is of type i; $i = 1, \dots I$.

$$E[K_B] = \sum_{i=1}^{I} \alpha_i \left\{ \left[\theta_{RQ}(i) P_{RQ}(i) + \theta_{RI}(i) P_{RI}(i)\right] + \left[\theta_{RQ}(i)(1 - P_{BRQ}) + \theta_{RI}(i)(1 - P_{BRI})\right] E[K_B] \right\} \tag{3.5}$$

where $\theta_{RI}(i) = 1 - \theta_{RQ}(i)$.

Solving,

$$\begin{aligned} E[K_B] &= \frac{\sum_{i=1}^{I} \alpha_i \left[\theta_{RQ}(i) P_{RQ}(i) + \theta_{RI}(i) P_{RI}(i)\right]}{1 - \sum_{i=1}^{I} \alpha_i \left[\theta_{RQ}(i)(1 - P_{BRQ}) + \theta_{RI}(i)(1 - P_{BRI})\right]} \\ &= \frac{\sum_{i=1}^{I} \alpha_i \left[\theta_{RQ}(i) P_{RQ}(i) + \theta_{RI}(i) P_{RI}(i)\right]}{\sum_{i=1}^{I} \alpha_i \left[\theta_{RQ}(i) P_{BRQ} + \theta_{RI}(i) P_{BRI}\right]} \end{aligned} \tag{3.6}$$

Since $\dfrac{\theta_{RQ}(i) P_{RQ}(i) + \theta_{RI}(i) P_{RI}(i)}{\theta_{RQ}(i) P_{BRQ} + \theta_{RI}(i) P_{BRI}} \le \max\left[\dfrac{P_{RQ}(i)}{P_{BRQ}}, \dfrac{P_{RI}(i)}{P_{BRI}}\right]$ for $i = 1, \dots, I$, $0 \le \theta_{RQ}(i) \le 1$, it follows that Red would like to follow a strategy that maximizes the expected number of Blue kills; a convenient *heuristic* is analogous to (3.4), applied to individual target classes.

For a target of type i,

$$\begin{aligned} &\textit{Emit extensively if} \quad \frac{P_{RI}(i)}{P_{BRI}} > \frac{P_{RQ}(i)}{P_{BRQ}} \\ &\textit{Emit minimally if} \quad \frac{P_{RQ}(i)}{P_{BRQ}} > \frac{P_{RI}(i)}{P_{BRI}} \end{aligned} \tag{3.7}$$

The suggested heuristic policy for Blue that approximately minimizes the maximum value of $E[K_B]$ is to always present those targets of type i_B to Red where i_B is

$$i_B = \arg\min_i \left(\max\left(\frac{P_{RQ}(i)}{P_{BRQ}}, \frac{P_{RI}(i)}{P_{BRI}} \right) \right).$$

This model gives Red credit for being able to perfectly distinguish different types of Blue targets (certainly optimistic for Red), focusing on a priority list related to vulnerability of Red. It implicitly simply *omits* any attention by Red to valueless targets, such as decoys; prosecuting decoys both wastes the Red ammunition inventory and betrays Red presence. Subsequent models will rectify this simplification.

Another, and related, missing feature is the assumption that all Reds can correctly classify the different types of Blue Attackers. This unrealism may also be rectified.

Model S-3. Allowing for Red Misclassification of Blue Target Types

Suppose there are I Blue target types. Let α_i be the probability a Blue target is of type i, $i = 1, \ldots, I$. Let γ_{ij} be the probability Red classifies a Blue type i target as a type j target.

$$E[K_B]$$

$$= \sum_i \sum_j \alpha_i \gamma_{ij} \left\{ \theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i) + \left[\theta_{RQ}(j)(1 - P_{BRQ}) + \theta_{RI}(j)(1 - P_{BRI}) \right] E[K_B] \right\}$$

$$= \frac{\sum_i \sum_j \alpha_i \gamma_{ij} \left[\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i) \right]}{1 - \sum_i \sum_j \alpha_i \gamma_{ij} \left[\theta_{RQ}(j)(1 - P_{BRQ}) + \theta_{RI}(j)(1 - P_{BRI}) \right]} \tag{3.8}$$

$$= \frac{\sum_i \sum_j \alpha_i \gamma_{ij} \left[\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i) \right]}{\sum_i \sum_j \alpha_i \gamma_{ij} \left[\theta_{RQ}(j) P_{BRQ} + \theta_{RI}(j) P_{BRI} \right]}$$

To find the values of $\theta_{RQ}(j)$ that maximize $E[K_B]$, note that the value of $\theta_{RQ}(j)$ can be determined for each j independent of the other values. Fix the values of $\theta_{RQ}(i)$ $i \neq j$, then $E[K_B]$ can be rewritten as

$$g(\theta_{RQ}(j)) = \frac{c_1(0)+\sum_i \alpha_i \gamma_{ij}\left[\theta_{RQ}(j)P_{RQ}(i)+\theta_{RI}(j)P_{RI}(i)\right]}{c_2(0)+\sum_i \alpha_i \gamma_{ij}\left[\theta_{RQ}(j)P_{BRQ}+\theta_{RI}(j)P_{BRI}\right]}$$

$$= \frac{c_1(1)+\sum_i \pi(i|j)\left[\theta_{RQ}(j)P_{RQ}(i)+\theta_{RI}(j)P_{RI}(i)\right]}{c_2(1)+\sum_i \pi(i|j)\left[\theta_{RQ}(j)P_{BRQ}+\theta_{RI}(j)P_{BRI}\right]} \tag{3.9}$$

where

$$\pi(i|j) = \frac{\alpha_i \gamma_{ij}}{\sum_k \alpha_k \gamma_{kj}} \tag{3.10}$$

the conditional probability the target is of type i given it is classified as type j; $c_1(0)$, $c_2(0)$, $c_1(1)$, and $c_2(1)$ are constants not involving $\theta_{RQ}(j)$.

Let

$$f_Q(j) = \sum_i \pi(i|j)\frac{P_{RQ}(i)}{P_{BRQ}}$$

$$f_E(j) = \sum_i \pi(i|j)\frac{P_{RI}(i)}{P_{BRI}} \tag{3.11}$$

and

$$M(j) = \max\left(f_Q(j), f_E(j)\right). \tag{3.12}$$

Since

$$\frac{\sum_i \pi(i|j)\left[\theta_{RQ}(j)P_{RQ}(i)+\theta_{RI}(j)P_{RI}(i)\right]}{\sum_i \pi(i|j)\left[\theta_{RQ}(j)P_{BRQ}+\theta_{RI}(j)P_{BRI}\right]} \le M(j)$$

for $j = 1, \ldots, I$, $0 \le \theta_{RQ}(i) \le 1$, it follows that the heuristic/approximate strategy to maximize the expected number of Blue kills for Red is as follows:

For a target that is *classified* as type j

$$\textit{Emit extensively if} \quad \sum_{i=1}^{I} \pi(i|j)\frac{P_{RI}(i)}{P_{BRI}} > \sum_{i=1}^{I} \pi(i|j)\frac{P_{RQ}(i)}{P_{BRQ}}$$

$$\textit{Emit minimally if} \quad \sum_{i} \pi(i|j)\frac{P_{RI}(i)}{P_{BRI}} < \sum_{i} \pi(i|j)\frac{P_{RQ}(i)}{P_{BRQ}} \tag{3.13}$$

where (3.10) gives $\pi(i|j)$.

Finally, in an Appendix, we consider a more ambitious but again deterministic model that allows for presence of Blue *decoys* introduced to economically deceive Red into firing, and hence revealing itself.

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APPENDIX
MODEL D-2:
DETERMINISTIC INFORMATIONAL DYNAMICS WITH ATTACKER DECOYS PRESENT

Suppose that at time t after campaign initiation there are $B_{Ai}(t)$ active Blue Attackers in region $\mathfrak{R}$, within reach of any Reds (EAD units); let $B_{Si}(t)$ be the number of Blue attacker counterfeits (decoys or surrogates); these can attract Red missile shots and are vulnerable. Here $i = U$ or D, signifying undetected or detected.

Red States, and State Transition

Red EAD units are present in the region $\mathfrak{R}$ at $t = 0$; some can leave, and others can enter. Those within the region can either be *fixed in place* and potentially active against Blue intruders, or *in motion* from one location (hiding place and launching spot) to another.

Assume that when a Red is in motion it may be detected by Blue "overhead" assets, e.g. JSTARS or possibly satellites, but not immediately nor with certainty, *and* assume that such detections are corrupted by Red deliberate false targets/decoys and/or by involuntary false targets. Importantly, and as before, when a Red launches a missile against a Blue Attacker, it reveals itself and its location: if the anti-Blue missile is *quietly guided* (emission is used minimally) the probability of its detection is positive, but if the launcher *emits extensively* to perform guidance its presence is revealed with much higher probability. It is assumed that such Red (EAD) presence is recorded on Blue memory data bases and acted upon: the locations may be attacked, or observed and stimulated to fire again to reveal presence. Of course if the Red has moved, any Blue attack at a (former) location is likely to be useless (unless another Red has moved into the locality). Thus a Red unit can be in a *moving and undetected/detected* state, a *fixed and undetected/detected state*, with an additional recorded history of recent emission or none.

Model D-2

The following variables enumerate the numbers of Red EAD units in the various states at time t. By rights, these are discrete-valued (counts) random processes. But we describe them generically in terms that might be known, and unknown, to the Blue forces.

$R_{AFU}(t)$ = number of *active* Red units that are *fixed* in location and *undetected*

$R_{AFD}(t)$ = number of *active* Red units that are fixed in location and *detected.* These can have been detected by general Blue surveillance, e.g. ground observation or UAVs, or have revealed themselves from a recent missile shot at a Blue: either Attacker or Decoy. These Red units are subject to Blue attack/prosecution. They can change status by *moving*; during such a period they can be detected, possibly targeted, but cannot themselves launch missiles. Those that go into motion have obliterated the identity and location information given by a previous shot, especially one that utilized emission/illumination.

$R_{AMU}(t)$ = number of *active* Red units *in motion* and *undetected*;

$R_{AMD}(t)$ = number of *active* Red units *in motion* and *detected.*

Also define

$R_{SFU}(t)$, $R_{SFD}(t)$, $R_{SMU}(t)$, $R_{SMD}(t)$ = number(s) of *Red Decoys* (Red Surrogates) in the above categories at time t. Finally,

$R_{AK}(t)$ = number of active Reds killed by time t

$R_{SK}(t)$ = number of Red Decoys killed by time t

$B_{AU}(t)$ = number of undetected active Blues

$B_{AD}(t)$ = number of detected active Blues

$B_{SU}(t)$ = number of undetected Blue surrogates

$B_{SD}(t)$ = number of detected Blue surrogates

$B_{AK}(t)$ = number of active Blues killed by time t

Parameters:

$\lambda_R(t)$ (respectively $\lambda_B(t)$) = arrival rate of active Red (respectively Blue) shooters to area

$\lambda_{RS}(t)$ (respectively $\lambda_{BS}(t)$) = arrival rate of Red (respectively Blue) surrogates to area

β_M^{-1} = mean time an active Red shooter moves

β_F^{-1} = mean time a Red shooter stays in a fixed position

$p_{MF}(D, U)$ = probability a detected moving Red shooter that stops is lost from track

$\alpha(B|R)$ (respectively $\alpha(R|B)$) = rate at which a Red (respectively Blue) detected Blue (respectively Red) target is assigned to a Red (respectively Blue) shooter

θ_{RQ} (respectively θ_{BQ}) = probability a Red (respectively Blue) shooter emits minimally when shooting at a target (is quieter)

θ_{RI} (respectively θ_{BI}) = probability a Red (respectively Blue) shooter emits extensively when shooting at a Blue (respectively Red) target

$\delta_Q(R|B)$ (respectively $\delta_Q(B|R)$) = probability a quieter Red (respectively Blue) shooter is detected while shooting and put on Blue's (respectively Red's) targeting list

$\delta_I(R|B)$ (respectively $\delta_I(B|R)$) = probability an extensively-emitting Red (respectively Blue) shooter is detected while shooting and put on Blue's (respectively Red's) targeting list

$\delta_F(R|B)$ = probability an undetected fixed Red is detected by Blue

$\delta_M(R|B)$ = probability a moving Red is detected by Blue and put on his targeting list

$p_{QK}(B|R)$ (respectively $p_{QK}(R|B)$) = probability a minimally-emitting Red (respectively Blue) kills a Blue (respectively Red) target

$p_{EK}(B|R)$ (respectively $p_{EK}(R|B)$) = probability an extensively-emitting Red (respectively Blue) kills a Blue (respectively Red) target

$v_M(R|B)$ = rate at which a detected moving Red target is lost from track

$\nu(B|R)$ = rate at which a detected Blue target is lost by Red

$p_C(R|B)$ (respectively $p_C(B|R)$) = probability Blue (respectively Red) correctly classifies the detected Red (respectively Blue) target as active or surrogate

$$\frac{dR_{AFU}(t)}{dt} = \underbrace{\beta_M p_{MF}(D,U) R_{AMD}(t)}_{\substack{\text{rate at which detected}\\ \text{moving active Reds stop}\\ \text{and are lost when fixed}}} + \underbrace{\beta_M R_{AMU}(t)}_{\substack{\text{rate at which}\\ \text{undetected active}\\ \text{moving Reds stop}}} - \underbrace{\beta_F R_{AFU}(t)}_{\substack{\text{rate at which}\\ \text{undetected fixed}\\ \text{active Reds move}}}$$

$$- \left[\underbrace{\begin{array}{l} \alpha(B|R) R_{AFU}(t) \dfrac{B_{AD}(t)+B_{SD}(t)}{1+\left(B_{AD}(t)+B_{SD}(t)\right)} \dfrac{R_{AFU}(t)}{R_{AFU}(t)+R_{AFD}(t)} \\ \times\left(\theta_{RQ}\delta_Q(R|B)+\theta_{RI}\delta_I(R|B)\right) \end{array}}_{\text{rate at which detected Blue targets are assigned to undetected Red shooters and shooters are detected}} \right] \tag{A.1}$$

$$- \underbrace{\delta_F(R|B) p_C(R|B) R_{AFU}(t)}_{\substack{\text{rate at which undetected}\\ \text{fixed Reds are detected and}\\ \text{classified correctly}}}$$

$$\frac{dR_{AFD}(t)}{dt} = \left[\begin{array}{l} \alpha(B|R) \dfrac{R_{AFU}(t)}{R_{AFD}(t)+R_{AFU}(t)} R_{AFU}(t) \\ \times \dfrac{\left(B_{AD}(t)+B_{SD}(t)\right)}{1+\left(B_{AD}(t)+B_{SD}(t)\right)} \left(\theta_{RQ}\delta_Q(R|B)+\theta_{RI}\delta_I(R|B)\right) \end{array} \right] - \underbrace{\beta_F R_{AFD}(t)}_{\substack{\text{rate at which detected}\\ \text{fixed Reds move}}}$$

$$- \left[\underbrace{\begin{array}{l} \alpha(R|B)(B_{AD}(t)+B_{AU}(t)) \left(\dfrac{R_{AFD}(t)}{R_{AD}(t)+R_{SD}(t)} \right) \left(\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)} \right) \\ \times\left(\theta_{RQ} p_{QK}(R|B)+\theta_{RI} p_{EK}(R|B)\right) \end{array}}_{\text{rate at which detected active Reds are killed by Blues}} \right] \tag{A.2}$$

$$+ \underbrace{\beta_M R_{AMD}(t)\left(1-p_{MF}(D,U)\right)}_{\substack{\text{rate at which detected moving Reds}\\ \text{stop and are still detected}}} + \underbrace{\delta_F(R|B) p_C(R|B) R_{AFU}(t)}_{\substack{\text{rate at which Blue detects and}\\ \text{correctly classifies active Red targets}}}$$

where $R_{AD}(t) = R_{AFD}(t) + R_{AMD}(t)$ and $R_{SD}(t) = R_{SFD}(t) + R_{SMD}(t)$

$$\frac{dR_{AMD}(t)}{dt} = \lambda_R(t) + \underbrace{\delta_M(R|B)p_C(R|B)R_{AMU}(t)}_{\substack{\text{rate at which undetected active moving}\\ \text{Reds are detected and classified correctly}}} - \underbrace{\nu_M(R|B)R_{AMD}(t)}_{\substack{\text{rate at which detected}\\ \text{moving active Reds are lost}}} - \underbrace{\beta_M R_{AMD}(t)}_{\substack{\text{rate at which detected}\\ \text{moving active Reds}\\ \text{become fixed}}}$$

$$-\left[\underbrace{\begin{array}{l} \alpha(R|B)\left(B_{AD}(t)+B_{AU}(t)\right)\dfrac{R_{AMD}(t)}{R_{AD}(t)+R_{SD}(t)} \\ \times\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)}\left(\theta_{BQ}p_{QK}(R|B)+\theta_{BI}p_{EK}(R|B)\right) \end{array}}_{\text{rate at which detected moving active Reds are killed}}\right] \tag{A.3}$$

$$\frac{dR_{AMU}(t)}{dt} = -\underbrace{\delta_M(R|B)p_C(R|B)R_{AMU}(t)}_{\substack{\text{rate at which undetected moving active}\\ \text{Reds are detected and correctly classified}}} + \underbrace{\nu_M(R|B)R_{AMD}(t)}_{\substack{\text{rate at which detected}\\ \text{moving active Reds are lost}}}$$

$$+\underbrace{\beta_F\left(R_{AFU}(t)+R_{AFD}(t)\right)}_{\substack{\text{rate at which fixed active}\\ \text{Reds move}}} - \underbrace{\beta_M R_{AMU}(t)}_{\substack{\text{rate at which moving}\\ \text{active Reds stop}}} \tag{A.4}$$

$$\frac{dR_{SFU}(t)}{dt} = \underbrace{\beta_M R_{SMU}(t)}_{\substack{\text{rate at which}\\ \text{undetected moving Red}\\ \text{surrogates stop moving}}} - \underbrace{\beta_F R_{SFU}(t)}_{\substack{\text{rate at which}\\ \text{undetected fixed Red}\\ \text{surrogates start moving}}}$$

$$-\underbrace{\delta_F(R|B)R_{SFU}(t)}_{\substack{\text{rate at which undetected}\\ \text{fixed Red surrogates are}\\ \text{detected by Blue}}} + \underbrace{\beta_M p_{MF}(D,U)R_{SMD}(t)}_{\substack{\text{rate at which detected Red surrogates}\\ \text{become fixed and are lost}}} \tag{A.5}$$

$$\frac{dR_{SFD}(t)}{dt} = \underbrace{\delta_F(R|B)\big(1-p_C(R|B)\big)R_{SFU}(t)}_{\substack{\text{rate at which fixed}\\ \text{Red surrogates are detected and}\\ \text{incorrectly classified as active targets}}} + \underbrace{\beta_M\big(1-p_{MF}(D,U)\big)R_{SMD}}_{\substack{\text{rate at which detected moving}\\ \text{Red surrogates become fixed}\\ \text{and are not lost}}} - \beta_F R_{SFD}(t)$$

$$-\left[\underbrace{\begin{array}{l} \alpha(R|B)\big(B_{AD}(t)+B_{AU}(t)\big)\dfrac{R_{SFD}(t)}{R_{AD}(t)+R_{SD}(t)} \\ \times\left(\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)}\right)\big(\theta_{BQ}p_{QK}(R|B)+\theta_{BI}p_{EK}(R|B)\big) \end{array}}_{\text{rate at which fixed Red surrogates are killed}}\right] \tag{A.6}$$

$$\frac{dR_{SMU}(t)}{dt} = \lambda_{RS}(t) + \underbrace{\beta_F\big(R_{SFU}(t)+R_{SFD}(t)\big)}_{\substack{\text{rate at which fixed}\\ \text{Red surrogates start}\\ \text{to move}}} - \underbrace{\delta_M(R|B)R_{SMU}(t)}_{\substack{\text{rate at which undetected moving}\\ \text{Red surrogates are detected}\\ \text{by Blue}}} - \underbrace{\beta_M R_{SMU}(t)}_{\substack{\text{rate at which}\\ \text{undetected moving}\\ \text{Red surrogates}\\ \text{become fixed}}} \tag{A.7}$$

$$\frac{dR_{SMD}(t)}{dt} = \underbrace{\delta_M(R|B)\big(1-p_C(R|B)\big)R_{SMU}(t)}_{\substack{\text{rate at which undetected moving}\\ \text{Red surrogates are detected}\\ \text{by Blue and misclassed as being active}}} - \underbrace{\beta_M R_{SMD}(t)}_{\substack{\text{rate at which}\\ \text{detected moving Red}\\ \text{surrogates become fixed}}}$$

$$-\underbrace{\left[\begin{array}{l} \alpha(R|B)\big(B_{AD}(t)+B_{AU}(t)\big)\dfrac{R_{SMD}}{R_{AD}(t)+R_{SD}(t)} \\ \times\left(\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)}\right)\big(\theta_{BQ}p_{QK}(R|B)+\theta_{BI}p_{EK}(R|B)\big) \end{array}\right]}_{\text{rate at which Blue kills detected moving Red surrogates}} \tag{A.8}$$

$$\frac{dB_{AU}(t)}{dt} = \underbrace{\lambda_B(t)}_{\substack{\text{arrival rate}\\ \text{of Blue actives}\\ \text{to area}}} - \left[\underbrace{\begin{array}{l} \alpha(R|B)\dfrac{B_{AU}(t)}{B_{AU}(t)+B_{AD}(t)}\big(B_{AU}(t)+B_{AD}(t)\big)\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)} \\ \times\big(\theta_{BQ}\delta_Q(B|R)+\theta_{BI}\delta_E(B|R)\big) \end{array}}_{\text{rate at which shooting active Blues are detected by Red}}\right] \tag{A.9}$$

$$+\underbrace{\nu(B|R)B_{AD}(t)}_{\substack{\text{rate at which detected}\\ \text{active Blues are lost}}} - \underbrace{\delta_M(B|R)p_C(B|R)B_{AU}(t)}_{\substack{\text{rate at which undetected active Blues are}\\ \text{detected by Red and correctly classified}}}$$

$$\frac{dB_{AD}(t)}{dt}=-\left[\underbrace{\begin{array}{l}\alpha(B|R)\left(R_{AFU}(t)\dfrac{R_{AFU}(t)}{R_{AFU}(t)+R_{AFD}(t)}+R_{AFD}(t)\dfrac{R_{AFD}(t)}{R_{AFU}(t)+R_{AFD}(t)}\right)\\ \times\left(\dfrac{B_{AD}(t)+B_{SD}(t)}{1+B_{AD}(t)+B_{SD}(t)}\right)\left(\dfrac{B_{AD}(t)}{B_{AD}(t)+B_{SD}(t)}\right)\\ \times\left(\theta_{RQ}p_{QK}(R)+\theta_{RI}p_{EK}(R)\right)\end{array}}_{\text{rate at which detected active Blues are killed by Reds}}\right]$$

$$+\left[\underbrace{\alpha(R|B)\left(\begin{array}{l}\left(\dfrac{B_{AU}(t)}{B_{AU}(t)+B_{AD}(t)}\right)\left(B_{AU}(t)+B_{AD}(t)\right)\\ \times\left(\dfrac{R_{AD}(t)+R_{SD}(t)}{1+R_{AD}(t)+R_{SD}(t)}\right)\left(\theta_{BQ}\delta_Q(B|R)+\theta_{BI}\delta_E(B|R)\right)\end{array}\right)}_{\text{rate at which shooting undetected active Blues are detected by Red}}\right] \tag{A.10}$$

$$-\underbrace{\nu(B|R)B_{AD}(t)}_{\substack{\text{rate at which detected}\\ \text{active Blues are lost}}}+\underbrace{\delta(B|R)p_C(B|R)B_{AU}(t)}_{\substack{\text{rate at which undetected active Blues}\\ \text{are detected and correctly classified}}}$$

$$\frac{dB_{AK}(t)}{dt}=\left[\underbrace{\begin{array}{l}\alpha(B|R)\left(R_{AFU}(t)\dfrac{R_{AFU}(t)}{R_{AFU}(t)+R_{AFD}(t)}+R_{AFD}(t)\dfrac{R_{AFD}(t)}{R_{AFU}(t)+R_{AFD}(t)}\right)\\ \times\left(\dfrac{B_{AD}(t)+B_{SD}(t)}{1+B_{AD}(t)+B_{SD}(t)}\right)\left(\dfrac{B_{AD}(t)}{B_{AD}(t)+B_{SD}(t)}\right)\\ \times\left(\theta_{RQ}p_{QK}(R)+\theta_{RI}p_{EK}(R)\right)\end{array}}_{\text{rate at which detected Blues are killed by Reds}}\right] \tag{A.11}$$

$$\frac{dB_{SU}(t)}{dt}=\underbrace{\lambda_{BS}(t)}_{\substack{\text{arrival rate of}\\ \text{Blue surrogates}}}-\underbrace{\delta(B|R)B_{SU}(t)}_{\substack{\text{detection of Blue}\\ \text{surrogates by Red}}} \tag{A.12}$$

$$\frac{dB_{SD}(t)}{dt} = \underbrace{\delta(B|R)(1-p_C(B|R))B_{SU}(t)}_{\substack{\text{rate at which surrogate Blues}\\ \text{are detected and incorrectly}\\ \text{classified as active}}} - \underbrace{\left[\begin{array}{l} \alpha(B|R)\left(R_{AFU}(t)\dfrac{R_{AFU}(t)}{R_{AFU}(t)+R_{AFD}(t)}+R_{AFD}(t)\dfrac{R_{AFD}(t)}{R_{AFU}(t)+R_{AFD}(t)}\right) \\ \times\left(\dfrac{B_{AD}(t)+B_{SD}(t)}{1+B_{AD}(t)+B_{SD}(t)}\right)\left(\dfrac{B_{SD}(t)}{B_{AD}(t)+B_{SD}(t)}\right) \\ \times(\theta_{RQ}p_{QK}(R)+\theta_{RI}p_{EK}(R)) \end{array}\right]}_{\text{rate at which detected incorrectly classified Blue surrogates are killed}} \quad \text{(A.13)}$$

We do not explore these expressions numerically at this time, although such is within the capability of many differential equation solvers, such as MATLAB.

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